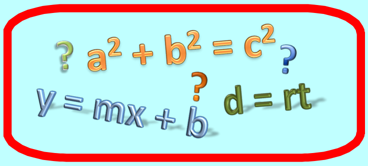
Math 115

Fall 2018

Lecture 29



Feb 19-8:47 AM

Use cross-multiplication to solve:

① $\frac{x}{5} = \frac{x+2}{8}$

$8x = 5(x+2)$

$8x = 5x + 10$

$8x - 5x = 10$

$3x = 10$

$\boxed{x = \frac{10}{3}}$

$\left\{ \frac{10}{3} \right\}$

② $\frac{5}{x-6} = \frac{x}{x-2}$ E.V. 2, 6

$x(x-6) = 5(x-2)$

$x^2 - 6x = 5x - 10$

$x^2 - 6x - 5x + 10 = 0$

$x^2 - 11x + 10 = 0$

$(x-10)(x-1) = 0$

By Z.F.P.

$x - 10 = 0$ $x = 10$ $\{1, 10\}$

$x - 1 = 0$ $x = 1$

Use LCD to clear fractions, then solve.
Be aware of excluded values.

$$\frac{1}{x+2} + \frac{1}{x-2} = \frac{4}{x^2-4}$$

LCD $= (x+2)(x-2)$, E.V. $= \pm 2$

$1(x-2) + 1(x+2) = 4$

$x - 2 + x + 2 = 4$

$2x = 4$

$x = 2$

$\varnothing$

Use LCD to clear fractions, then solve.
Be aware of excluded values.

$$\frac{2x+3}{x-1} - \frac{2}{x+3} = \frac{5-6x}{x^2+2x-3}$$

LCD $= (x+3)(x-1)$ E.V.: $1, -3$

$(x+3)(2x+3) - (x-1)\cdot 2 = 5 - 6x$

$2x^2 + 3x + 6x + 9 - 2x + 2 - 5 + 6x = 0$

$2x^2 + 13x + 6 = 0$

$121 = 11^2$

$a=2$ $b=13$ $c=6$

$b^2-4ac = 13^2 - 4(2)(6) = 169 - 48 = 121$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-13 \pm \sqrt{121}}{2(2)} = \frac{-13 \pm 11}{4}$$

$\left\{-6, -\frac{1}{2}\right\}$

$x = \frac{-13+11}{4} = \frac{-2}{4} = \boxed{\frac{-1}{2}}$ $x = \frac{-13-11}{4} = \frac{-24}{4} = \boxed{-6}$

John can do a certain job in 3 hrs alone while Tom can do the same job in 6 hrs alone.
How long does it take for them working together to do this job?

John → 3 hrs → Rate $\frac{1}{3}$

Tom → 6 hrs → Rate $\frac{1}{6}$

Work by John + Work by Tom = one Complete work

Rate · Time   Rate · Tim

2 hrs

$$\frac{1}{3}\cdot t + \frac{1}{6}\cdot t = 1$$

$$\frac{t}{3} + \frac{t}{6} = 1$$

LCD = 6

$$2t + 1t = 6$$

$$3t = 6$$

$$\boxed{t = 2}$$

It takes Raul 5 minutes longer than Iris to do a job.
working together, they can do it in 6 minutes.
Find how long it takes Raul to do the job alone.

Iris → x minutes → Rate $\frac{1}{x}$

Raul → $(x+5)$ minutes → Rate $\frac{1}{x+5}$

Work by Iris + Work by Raul = one complete work

Rate · Time   Rate · Time

$$\frac{1}{x}\cdot 6 + \frac{1}{x+5}\cdot 6 = 1 \Rightarrow \frac{6}{x} + \frac{6}{x+5} = 1$$

$$\frac{6}{x} + \frac{6}{x+5} = 1$$

$LCD = x(x+5)$, $x > 0$

$6(x+5) + 6x = x(x+5)$

$6x + 30 + 6x = x^2 + 5x$

$12x + 30 = x^2 + 5x$

$x^2 + 5x - 12x - 30 = 0$

$x^2 - 7x - 30 = 0$

$(x-10)(x+3) = 0$

$\rightarrow x = 10$, $\cancel{x = -3}$

Iris $\rightarrow$ 10 mins.

Raul $\rightarrow$ 15 minutes

Pipe A can fill up an empty Pool in 8 hrs.

Pipe B - - - - - . . . - - - - - in 10 hrs.

How long does it take both pipes working to fill up an empty pool?

$$\frac{1}{8} \cdot t + \frac{1}{10} \cdot t = 1$$

$$\frac{t}{8} + \frac{t}{10} = 1$$

$LCD = 40$

$5t + 4t = 40$

$9t = 40$

$t = \frac{40}{9} \approx 4.4$

About 4.4 hrs

Pipe A can fill up an empty Pool in 8 hrs.
Pipe B - - empty . . . full Pool in 10 hrs.
How long does it take both pipes working to fill up an empty pool?

$$\frac{1}{8}\cdot t - \frac{1}{10}\cdot t = 1$$

$$\frac{t}{8} - \frac{t}{10} = 1$$

LCD = 40

$$5t - 4t = 40$$

$$t = 40$$

40 hrs

Distance Problems

$d = r \cdot t$

t → Time

r → Rate (speed)

d → Distance

You drive at 60 mph for 4.5 hrs,

$d = r \cdot t$

$= 60 \cdot (4.5) = 270$

270 miles

$d = r \cdot t$ → Solve for t

$$t = \frac{d}{r} \qquad t = \frac{270}{60}$$

$t = 4.5$

Raul can run 15 miles in the same time that Iris can do 21 miles.

Raul runs 2 mph slower than Iris.

Find speed for both.

	r	t	d
Raul	$x-2$	t	15
Iris	x	t	21

$t_{Raul} = t_{Iris}$

$$\frac{15}{x-2} = \frac{21}{x}$$

Cross-multiply

$21(x-2) = 15x$

$21x - 42 = 15x$

$21x - 15x = 42$

$6x = 42$

$\boxed{x = 7}$

Iris 7 mph

Raul 5 mph

Alex drives 210 miles in the same time that Luis drives 150 miles

Alex speed is 30 mph slower than twice of speed by Luis.

Find speed by both.

	r	t	d
Alex	$2x-30$	t	210
Luis	x	t	150

$t_{Alex} = t_{Luis}$

$$\frac{\cancel{210}^{21}}{2x-30} = \frac{\cancel{150}^{15}}{x}$$

$$\frac{\cancel{21}^{7}}{2x-30} = \frac{\cancel{15}^{5}}{x}$$

$5(2x-30) = 7x$

$10x - 150 = 7x$

$10x - 7x = 150$

$3x = 150$

$x = 50$

Luis → 50 mph

Alex → 70 mph

Wind is blowing at 40 mph.

A plane flies 960 miles with wind in the same time that it flies 640 miles against the wind.

Find speed of the plane in still air.

	r	t	d
with wind	$x+40$	t	960
Against wind	$x-40$	t	640

$$t_{with} = t_{Against}$$

$$\frac{\cancel{960}^{\cancel{96}^{3}}}{x+40} = \frac{\cancel{640}^{\cancel{64}^{2}}}{x-40}$$

Solve $\frac{3}{x+40} = \frac{2}{x-40}$

$3(x-40) = 2(x+40)$

$3x - 120 = 2x + 80$

$3x - 2x = 80 + 120$

$x = 200$

200 mph

A boat travels 165 miles downstream in the same time that it travels 135 miles upstream. Speed of the current is 5 mph.

Find speed of the boat in still water.

	r	t	d
downstream	$x+5$	t	165
upstream	$x-5$	t	135

$$t_{upstream} = t_{downstream}$$

$$\frac{\cancel{135}^{\cancel{27}^{9}}}{x-5} = \frac{\cancel{165}^{\cancel{33}^{11}}}{x+5}$$

$11(x-5) = 9(x+5)$

$11x - 55 = 9x + 45$

$11x - 9x = 45 + 55$

$2x = 100$

$\boxed{x = 50}$

50 mph

Marisela drove 90 miles in the city and 130 miles on the Highway.
She was driving 20 mph faster on the highway.
She drove a total of 4 hrs.
Find speed on the highway.

$t_1 + t_2 = 4$

$$\frac{90}{x} + \frac{130}{x+20} = 4$$

$x > 0$

	r	t	d
City	x	t_1	90
Highway	$x+20$	t_2	130

$$\frac{90}{x} + \frac{130}{x+20} = 4$$

LCD $= x(x+20)$

$x > 0$

↳ time

$90(x+20) + 130x = 4x(x+20)$

Divide by 2

$45(x+20) + 65x = 2x(x+20)$

$45x + 900 + 65x = 2x^2 + 40x$

$110x + 900 = 2x^2 + 40x$

$2x^2 + 40x - 110x - 900 = 0$

$2x^2 - 70x - 900 = 0$

Divide by 2 to reduce

$x^2 - 35x - 450 = 0$

$(x-45)(x+10) = 0$

45

$\cancel{-10}$

45 mph in the city

65 mph on the highway

Brandon trains for a total of 6 hrs/day on the local beach.
He swims 3 miles against the current, and 33 miles with the current.
Speed of the current is 5 mph.
How fast can he swim in still water?

$t_1 + t_2 = 6$

$\frac{\cancel{33}^{11}}{x+5} + \frac{\cancel{3}^{1}}{x-5} = \cancel{6}^{2}$

$\frac{11}{x+5} + \frac{1}{x-5} = 2$

$x > 5$

LCD $= (x+5)(x-5)$

$11(x-5) + 1(x+5) = 2(x+5)(x-5)$

	r	t	d
with	$x+5$	t_1	33
against	$x-5$	t_2	3

$11(x-5) + 1(x+5) = 2(x+5)(x-5)$

$11x - 55 + x + 5 = 2(x^2 - 25)$

$12x - 50 = 2(x^2 - 25)$

Divide by 2 to reduce

$6x \cancel{-25} = x^2 \cancel{-25}$

$x^2 - 6x = 0$

$x(x-6) = 0$

$\cancel{x = 0}$ $\boxed{x = 6}$

6 mph